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The GS Type IIB superstring action on $AdS_3 \times S_3 \times T^4$

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Abstract: We describe the type IIB supergravity background on $AdS_3 \otimes S_3 \otimes T^4$ using the potentials of $AdS_{3|4} \otimes S_3 \otimes T^4$ and we use the supersolvable algebra associated to $AdS_{3|4}$ to compute the κ gauge fixed type IIB string action. From the explicit form of the action we can clearly see how passing from pure NSNS backgrounds to pure RR backgrounds the WZW term disappears.

Keywords: Superstrings and Heterotic Strings, Superstring Vacua, Extended Supersymmetry.

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1. Introduction

The AdS/CFT conjecture [1, 2, 3] has recently attracted a lot of interest and work, one of its consequences is that the classical (i.e. no string loops) type IIB superstring propagating in a $AdS_5 \otimes S_5$ background is the masterfield of the $\mathcal{N} = 4$ $D = 4$ SYM based on the $U(N)$ gauge group. It has therefore become important to write the classical action for the above mentioned type IIB superstring and then to first quantize it. The first step has been accomplished in a series of works [7, 8, 9] which culminated in two different proposals [4, 5] which were shown to be equivalent [6].

Even if the resulting action can be very easily written down and we can thrust it is the proper conformal action [7, 10], it is unclear how to proceed to quantize it explicitly, even in a simplified but equivalent form [11], the main reason being it describes a RR background which reflects into the fact that the theory is not a usual WZW model because it has not the left/right symmetry [7]. In view of the fact that explicit computations of $\mathcal{N}=4$ $D=4$ SYM amplitudes in the leading order [12] and next to leading order [13] in λ ($\lambda = N g_{YM}^2 \gg 1$) based on this conjecture have revealed the possible appearance of logarithm and even if the results in this field are far from be definitive because of the difficulties of computing all the contributions explicitly, it has become even more important to be able to understand how to quantize this action.

In order to get some hint on how to proceed to quantize such a kind of action we could start to look at some simpler examples where one can at least partially compare with well defined NSR formulations: it is in fact purpose of this paper to write the explicit κ gauge fixed action for type IIB string propagating on $AdS_3 \otimes S_3 \otimes T^4$ whose NSR formulation has been worked out in [14]. This non dilatonic background can be pure NSNS, pure RR or a mixing of the two since it can be thought as generated by a dionic string in the type IIB supergravity dimensional reduced to $D = 6$: unfortunately it turns out that the two backgrounds are so different to make their comparison quite difficult.

The plan of the paper is the following.

In section 2 we compute the supergravitational background corresponding to a spontaneous compactification on $AdS_3 \otimes S_3 \otimes T^4$ and we express the background fields, thought as fields living in a superspace, through the fields of the superspace $AdS_{3|4}$ and the vielbein and spin-connection of $S_3 \otimes T^4$.

In section 3 we derive the form of the background fields when fixing the κ symmetry using the supersolvable approach.

In section 4 we give the action for the type IIB superstring propagating in this background, we discuss and comment the form of the action.

In a series of appendices we give more details on the computations and we prove that the proposed action actually fixes the κ symmetry as claimed.

2. The supergravitational background

We will now construct the supergravitational background explicitly: this will be performed in three steps

1. we solve the equations of motion and we get the field strengths which are associated with the spontaneous compactification on $AdS_3 \otimes S_3 \otimes T^4$;

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2. since the fermions are set to zero we look at their supersymmetric transformations and we determine the form and the number of Killing vectors. Their form suggests the ansatz for the gravitino in the next point;

3. we finally express all the fields through the generalized potentials of $SU(1,1|2,0) \otimes SU(1,1|0,2)$ and the vielbein and spin connection of $S_3 \otimes T^4$: in this way all the fields become defined on the superspace $AdS_{3|4}$.

2.1 The spontaneous compactification on $AdS_3 \otimes S_3 \otimes T^4$

Since we want a solution of the equations of motion which has a spontaneous compactification on $AdS_3 \otimes S_3 \otimes T^4$, it is natural to make the following ansatz which in the notation of [15][1] reads

$$
\begin{aligned}
R^{ab}_{\,..\,cd} &= r_1 \delta^{ab}_{cd} \qquad R^{ij}_{\,..\,kl} = r_2 \delta^{ij}_{kl} \qquad R^{rs}_{\,..\,tu} = 0 && (2.1)\\
G_{abc} &= g_1 \epsilon_{abc} && (2.2)\\
G_{ijk} &= g_2 \epsilon_{ijk} && (2.3)\\
V^{\alpha}_{\pm} &= const \quad \Rightarrow \quad Q = 0\,, \quad P_a = 0 && (2.4)\\
F_{\widehat{a}_1 \ldots \widehat{a}_5} &= 0 && (2.5)\\
\rho_{\widehat{a}\widehat{b}} &= 0 && (2.6)\\
\lambda &= 0 \quad \Rightarrow \quad D_{\widehat{a}} \lambda = 0\,, && (2.7)
\end{aligned}
$$

where $\widehat{a}, \widehat{b}, \ldots \in \{0, \ldots, 9\}$, $a, b, \ldots \in \{0,1,2\}$, $u, v, \ldots \in \{3, \ldots, 9\}$, $i, j, \ldots \in \{3,4,5\}$, $r, s, \ldots \in \{6, \ldots, 9\}$ (see appendix A for further conventions) and $G_{\widehat{a}\widehat{b}\widehat{c}} = \epsilon_{\alpha\beta} V^{\alpha}_{+} F^{\beta}_{\widehat{a}\widehat{b}\widehat{c}}$.

It is then a simple matter to solve the equation of motions and to find that

$$
-r_1 = r_2 = \mid e \mid^2\,, \qquad g_1 = \frac{2}{3} e\,, \qquad g_2 = \frac{2}{3} e\rho \quad (\rho = \pm 1)\,, \tag{2.8}
$$

where e is an arbitrary complex number and to verify that the complex tensor $G_{\widehat{a}\widehat{b}\widehat{c}}$ is covariantly constant in this background, i.e. $D^{\widehat{a}} G_{\widehat{a}\widehat{b}\widehat{c}} = 0$. We notice that the constraint $g_1^2 = g_2^2$ is a consequence of the equation of motion for the scalars.

2.2 The Killing vectors

As a first step we write the supersymmetric transformation rules for the fermions in this background

$$
\begin{aligned}
\delta\lambda &= -\frac{i}{8} \Gamma^{\widehat{a}\widehat{b}\widehat{c}} \widehat{\epsilon}\, G_{\widehat{a}\widehat{b}\widehat{c}}\,, && (2.9)\\
\delta\Psi &= D\widehat{\epsilon} + \frac{1}{32} \left(9\, \Gamma^{\widehat{a}\widehat{b}} V^{\widehat{c}} + \Gamma^{\widehat{a}\widehat{b}\widehat{c}\widehat{d}} V_{\widehat{d}} \right) \widehat{\epsilon}_c\, G_{\widehat{a}\widehat{b}\widehat{c}}\,, && (2.10)
\end{aligned}
$$

[1]With the minor modifications given by $\psi \to \Psi$, $\psi^* \to \Psi_c \equiv \widehat{C}\overline{\Psi}^T$ and $\varpi^{ab} \to \Omega^{\widehat{a}\widehat{b}}$.

where $\widehat{\epsilon}_c = \widehat{C}\,\widehat{\epsilon}^T$ is the charge conjugate of the infinitesimal parameters $\widehat{\epsilon}$ and all the field strengths are restricted to the spacetime.

Since the gravitino transformation rule involves both the parameters $\widehat{\epsilon}$ and its charge conjugate $\widehat{\epsilon}_c$ the only way to set to zero $\delta\Psi$ is to use a parameter $\widehat{\epsilon}$ which is a sum of linearly independent Majorana spinors, thus breaking at least half of the number of local charges, explicitly:

$$\widehat{\epsilon} = \begin{pmatrix} 0_{16} \\ e^{i\varphi_N}\ \epsilon_N \otimes \widetilde{\eta}^N \end{pmatrix}, \tag{2.11}$$

where $\epsilon_N = C\,\overline{\epsilon}_N$ is a set of N (N to be determined) AdS_3 Majorana spinors and $\widetilde{\eta}^N = \widetilde{C}\,\widetilde{\overline{\eta}}^N$ is a set of N $S_3 \otimes T^4$ Majorana spinors.

The dilatino variation implies that

$$1_2 \otimes \overline{\gamma}_5\,\widetilde{\eta}^N = \rho\,\widetilde{\eta}^N\,, \tag{2.12}$$

where the sign ρ is the same as the one which enters eq. (2.8).

The gravitinos variation yields more constraints and suggests how to express the gravitino (2.26), when thought as a field on the superspace, using the $AdS_{3|4}\otimes S_3 \otimes T^4$ generalized potentials. Explicitly we get (we denote Ψ_- the 16 lower components of the antiWeyl gravitino Ψ)[2]

$$\begin{aligned}\delta\Psi_- = e^{i\varphi_N}&\left(D_{AdS_3}\,\epsilon_N + i\frac{e}{2}e^{-2i\varphi_N}\ \gamma_a\epsilon_N\,V^a\right)\otimes\widetilde{\eta}^N + \\ &+ e^{i\varphi_N}\,\epsilon_N \otimes \left(D_{S_3\otimes T^4}\,\widetilde{\eta}^N - \frac{e}{2}\rho e^{-2i\varphi_N}\ \gamma_i \otimes 1_4\widetilde{\eta}^N\,V^i\right),\end{aligned}$$

which can be solved as

$$\left(D_{AdS_3}\,\epsilon_N - i\,s_N\,\frac{|e|}{2}\,\gamma_a\epsilon_N\,E^a\right) = 0\,, \tag{2.13}$$

$$\left(D_{S_3\otimes T^4}\,\widetilde{\eta}^N + s_N\,\frac{|e|}{2}\,\widetilde{\gamma}_i\widetilde{\eta}^N\,E^i\right) = 0\,, \tag{2.14}$$

where

$$-\frac{e}{|e|}\,e^{-2i\varphi_N} = s_N = \pm 1\,, \tag{2.15}$$

because of the integrability conditions of the two equations (2.13), (2.14). Notice that in eqs. (2.13), (2.14) we have explicitly used the fact that the restriction of the zehnbein to the spacetime gives the classical zehnbein, i.e. $V^{\widehat{a}}|_{''\Theta''=0} = E^{\widehat{a}}$.

We will denote a $S_3 \otimes T^4$ spinor $\widetilde{\eta}^N$ which satisfies eq. (2.14) with the substitution $s_N \to -s$ as $\widetilde{\eta}^N_{(s)}$ in order to stress the relative sign of the two terms of the equation (2.14).

[2] We also define $D_{AdS_3}\,\epsilon = \left(d - \frac{1}{4}\omega^{ab}\gamma_{ab}\right)\epsilon$ and $D_{S_3\otimes T^4}\,\widetilde{\eta} = \left(d - \frac{1}{4}\omega^{ij}\widetilde{\gamma}_{ij}\right)\widetilde{\eta}$ where ω are the classical spin-connections, i.e. without Θ dependence.

Before counting the number of real supersymmetric charges surviving in this background and being able to specify in which range the index N runs, we need to better understand how $\tilde{\eta}^N$can be decomposed w.r.t. T^4 and S_3 spinors. It is not difficult to see that the Majorana $\tilde{\eta}^N$spinors can be decomposed as

$$\tilde{\eta}^N_{(s)} \equiv \tilde{\eta}^{(n,m)}_{(s)} = H^N_{pq}\, \eta^p_{(s)} \otimes \kappa^q \,, \qquad H^N = \sigma_2 H^*_N \sigma_2 \,, \tag{2.16}$$

where the constraint on H_N is due to the Majorana condition which $\tilde{\eta}^N_{(s)}$ have to satisfy and to the fact that

- κ^q $(q = 1, 2)$ are a couple of constant chiral symplectic-Majorana spinors on the 4-torus

$$\overline{\gamma}_5\, \kappa^q = \rho\, \kappa^q \,, \qquad \kappa_{c\,q} \equiv \overline{C}\overline{\kappa}^T_q = \epsilon_{qp}\, \kappa^p \,, \qquad \overline{\kappa}_p \kappa^q = \delta^q_p \,, \tag{2.17}$$

  with $\epsilon_{12} = 1$ and

- $\eta^p_{(s)}$ $(p = 1, 2)$ are (for any s) a couple of symplectic-Majorana spinors on the 3-sphere

$$\eta_{c\,p} \equiv C\, \overline{\eta}^T_p = \epsilon_{pq}\, \eta^q \,, \qquad \overline{\eta}_p \eta^q = \delta^q_p \,, \tag{2.18}$$

  which satisfy the killing equation on the 3-sphere

$$D_{S_3} \eta^p_{(s)} - s\frac{|e|}{2} \rho\, \gamma_i \eta^p_{(s)}\, E^i = 0 \,. \tag{2.19}$$

It is now obvious that both p and q in (2.16) can take two values and that the index $N \equiv (n, m)$ can take 4 values as we can immediately find 4 matrices which satisfy the second equation in eq. (2.16) H: $\{1,\ i\sigma_1,\ i\sigma_2,\ i\sigma_3\}$, hence the number of real supersymmetric charges is $2 \cdot 2 \cdot 4 = 16$ where the first factor 2 accounts for the two possible values of s_N (2.15) and the second factor is the number of real components of ϵ_N.

2.3 The type IIB fields as fields living on the superspace $AdS_{3|4} \otimes S_3 \otimes T^4$

We are now ready to write our ansatz for how to express the supergravity fields leaving on $D = 10$ superspace using those defined on $AdS_{3|4} \otimes S_3 \otimes T^4$. More precisely we make use of

- the fields of the super anti de Sitter in 3D and 4 complex charges $AdS_{3|4} = \frac{SU(1,1|2,0)\otimes SU(1,1|0,2)}{SO(1,2)\otimes O(4)} \left\{\omega^{(\pm)a}(x, \Theta^{(\pm)}),\ A^{(\pm)n}_{.m}(x, \Theta^{(\pm)}),\ \chi^{(\pm)}_n(x, \Theta^{(\pm)})\right\}$ (see appendices B and C for a complete explanation of the notations and derivation of these results) which satisfy

$$d\omega^{(\pm)a} + \frac{1}{2}\epsilon^a_{.\,bc}\, \omega^{(\pm)b}\omega^{(\pm)c} - i s^{(\pm)}\, \overline{\chi}^{(\pm)n}\gamma^a\chi^{(\pm)}_n = 0 \,, \tag{2.20}$$

$$dA^{(\pm)n}_{.m} + A^{(\pm)n}_{.p}\, A^{(\pm)p}_{.m} - s^{(\pm)} \left(\overline{\chi}^{(\pm)n} \chi^{(\pm)}_m - \frac{1}{2}\delta^n_m\, \overline{\chi}^{(\pm)p} \chi^{(\pm)}_p \right) = 0\,, \tag{2.21}$$

$$dB^{(\pm)} - s^{(\pm)} \frac{1}{2}\, \overline{\chi}^{(\pm)p} \chi^{(\pm)}_p = 0\,, \tag{2.22}$$

$$d\chi^{(\pm)}_n - \frac{i}{2} \omega^{(\pm)a}\, \gamma_a \chi^{(\pm)}_n + \chi^{(\pm)}_m\, A^{(\pm)m}_{.n} = 0 \tag{2.23}$$

and depend on x and $\Theta^{(\pm)}_n$ which are respectively the AdS_3 coordinates and a set of 4 complex Grassman coordinates which transform as two AdS_3 complex spinors and *not* as $AdS_3 \otimes S_3 \otimes T^4$ spinors; we notice moreover the presence of the arbitrary signs $s^{(\pm)}$ which seem not to be reabsorbable in a field redefinition once the AdS_3 γ matrices and the metric are given;

- the classical dreibein and spin-connection of S_3 ($E^i(y)$, $\omega^{ij}(y)$) which satisfy

$$dE^i - \varpi^i_{.j} E^j = 0\,, \tag{2.24}$$

$$d\omega^{ij} - \omega^{ik} \omega_k^{.j} = |e|^2 E^i E^j\,, \tag{2.25}$$

  and depend on the S_3 coordinates y;

- the classical vierbein of T^4 $E^r = dz^r$ where z^r are the T^4 coordinates.

When we take in account the form of the killing spinors (2.11) the ansatz for the gravitino reads:

$$\begin{aligned} \Psi &= \kappa_3 \begin{pmatrix} 0 \\ e^{i\varphi_{(+)}} \chi^{(+)}_{(n,m)}(x,\Theta) \otimes \widetilde{\eta}^{(n,m)}_{(s)} + e^{i\varphi_{(-)}} \chi^{(-)}_{(n,m)}(x,\Theta) \otimes \widetilde{\eta}^{(n,m)}_{(t)} \end{pmatrix} = \\ &= \kappa_3 \begin{pmatrix} 0 \\ e^{i\varphi_{(+)}} \left(\chi^{(+)}_n(x,\Theta) \otimes \widetilde{\eta}^n_{(s)c} + \chi^{(+)n}_c(x,\Theta) \otimes \widetilde{\eta}_{(s)n} \right) \end{pmatrix} + \\ &+ \kappa_3 \begin{pmatrix} 0 \\ e^{i\varphi_{(-)}} \left(\chi^{(-)}_n(x,\Theta) \otimes \widetilde{\eta}^n_{(t)c} + \chi^{(-)n}_c(x,\Theta) \otimes \widetilde{\eta}_{(t)n} \right) \end{pmatrix}, \end{aligned} \tag{2.26}$$

where we have defined implicitly the Majorana component of χ_n as

$$\chi_n = \frac{1}{\sqrt{2}} \left(\chi_{(n,1)} + i\, \chi_{(n,2)} \right), \tag{2.27}$$

$$\chi^n_c = \frac{1}{\sqrt{2}} \left(\chi_{(n,1)} - i\, \chi_{(n,2)} \right),$$

and inversely we have defined the complex spinors $\widetilde{\eta}$ out of the Majorana ones $\widetilde{\eta}_N$ (defined in eq. (2.16)) as

$$\widetilde{\eta}_n = \frac{1}{\sqrt{2}} \left(\widetilde{\eta}_{(n,1)} + i\, \widetilde{\eta}_{(n,2)} \right) = (H_n)_{pq}\, \eta^p \otimes \kappa^q \tag{2.28}$$

and their charge conjugate $\widetilde{\eta}^n_c$. Moreover the Majorana killing spinors $\widetilde{\eta}_{(s)N} = \widetilde{\eta}_{(s)(m,l)}$ (2.16) are normalized as

$$\widetilde{\bar{\eta}}_{(s)N}\widetilde{\eta}_{(s)M} = \delta_{NM}\,. \tag{2.29}$$

Now that we have a set of complex C-number spinors $\widetilde{\eta}_n$ with the proper index structure we can write the ansatz for the remaining fields:

$$V^a = E^a(x,\Theta) = \frac{1}{2|e|}\left(\omega^{(+)a} - \omega^{(-)a}\right), \tag{2.30}$$

$$\begin{aligned} V^i &= E^i(y) + \kappa_1^{(+)}\,\widetilde{\bar{\eta}}^m_{(s)}(y)\,\widetilde{\gamma}^i\,\widetilde{\eta}_{(s)n}(y)\,A^{(+)n}_{\;\;.\,m}(x,\Theta) + \\ &\quad + \kappa_1^{(-)}\,\widetilde{\bar{\eta}}^m_{(t)}(y)\,\widetilde{\gamma}^i\,\widetilde{\eta}_{(t)n}(y)\,A^{(-)n}_{\;\;.\,m}(x,\Theta)\,, \end{aligned} \tag{2.31}$$

$$V^r = dz^r\,, \tag{2.32}$$

$$\Omega^{ab} = \omega^{ab}(x,\Theta) = \frac{1}{2}\epsilon^{abc}\left(\omega_c^{(+)} + \omega_c^{(-)}\right), \tag{2.33}$$

$$\begin{aligned} \Omega^{ij} &= \omega^{ij}(y) + \kappa_2^{(+)}\,\widetilde{\bar{\eta}}^m_{(s)}(y)\,\widetilde{\gamma}^{ij}\,\widetilde{\eta}_{(s)n}(y)\,A^{(+)n}_{\;\;.\,m}(x,\Theta) + \\ &\quad + \kappa_2^{(-)}\,\widetilde{\bar{\eta}}^m_{(t)}(y)\,\widetilde{\gamma}^{ij}\,\widetilde{\eta}_{(t)n}(y)\,A^{(-)n}_{\;\;.\,m}(x,\Theta)\,, \end{aligned} \tag{2.34}$$

$$\Omega^{rs} = 0\,. \tag{2.35}$$

In order to find the solution we have to fix the coefficients $\kappa_1^{(\pm)}$, $\kappa_2^{(\pm)}$, κ_3 ($\kappa_3 > 0$ since its phase is given by $\varphi_{(+)}$) and the phases $\varphi_{(\pm)}$; to determine the matrices H^N (2.16) and the sign s and t; to find to which form of the superalgebra $SU(1,1|2)$ the $\omega^{(\pm)a}(x,\Theta)$, $A^{(\pm)n}_{.m}(x,\Theta)$, $\chi_n^{(\pm)}(x,\Theta)$ belong, i.e. which signs $s^{(\pm)}$ have to be chosen in the Maurer-Cartan system (2.20-2.23).

Details on the actual computations are given in app. D, here we limit ourselves to illustrate the results.

Examining the pure gravitational Bianchi identities using these parametrization (2.26), (2.30)–(2.35) we get that

$$\kappa_1^{(\pm)} = \pm\frac{1}{2|e|}\,, \qquad \kappa_2^{(\pm)} = \frac{1}{2}\,, \qquad \kappa_3 = \frac{1}{2\sqrt{|e|}}\,, \tag{2.36}$$

$$s = -t = -1\,, \qquad s^{(\pm)} = \pm 1\,, \qquad e^{i(\varphi_{(-)}-\varphi_{(+)})} = i\,, \tag{2.37}$$

$$H_1 = \begin{pmatrix} & 0 \\ -1 & \end{pmatrix}, \qquad H_2 = \begin{pmatrix} 1 & \\ & 0 \end{pmatrix}, \tag{2.38}$$

while the integrability of the 2-form A^α equations requires

$$e^{2i\varphi_{(+)}} = -\frac{e}{|e|}\,. \tag{2.39}$$

We do not derive the explicit form of A^α here because of its complication but we will find its explicit expression after we have the κ fixed expression for the zehnbein and the gravitini in the next section.

3. Fixing the κ symmetry using the supersolvable and the κ gauge fixed expression for the supergravity background fields

In order to find the supersolvable algebras associated with the two forms of $SU(1,1|2)$, we look at the solvable algebra associated to $AdS_3 = \frac{SO(2,2)}{SO(1,2)} = \frac{SO(1,2)\otimes SO(1,2)}{SO(1,2)}$; from [16] we see that the generators of AdS_3 can be chosen to be

$$solv(AdS_3) = \{M_{23}, M_{+0}, M_{+1}\}\,,$$

where $M_{\mu\nu}$ are the $SO(2,2)$ generators and $M_{+\mu} = \frac{1}{\sqrt{2}}\left(M_{2\mu} + M_{3\mu}\right)$ (see appendix B for further details and more precise notations). We can now see how these generators can be written in terms of the generators $J_c^{(\pm)} = \epsilon_c^{\ ab}\frac{1}{4}\left(M_{ab} \pm \frac{1}{2}\epsilon_{ab\mu\nu}\, M^{\mu\nu}\right)$ of the two $SO(1,2)$

$$solv(AdS_3) = \left\{J_2^{(+)} - J_2^{(-)}\,,\, -J_+^{(+)} \pm J_-^{(-)}\right\}. \tag{3.1}$$

This equation (3.1) can be read in the dual space by saying that the bosonic part of the supersolvable algebra has to be generated by $\left\{\omega^{(+)2} - \omega^{(-)2}\,,\, \omega^{(+)+} \pm \omega^{(-)-}\right\}$. We write down the Maurer-Cartan equations for the two $SU(1,1|2)$ and we ask for consistency when we set either $\omega^{(+)-}$ or $\omega^{(-)+}$. If we generically denote the two components of the fermionic 1-form χ as

$$\chi = \begin{pmatrix} \zeta \\ \xi \end{pmatrix},$$

then it easy to find that the $(+)$ algebra reads

$$d\omega^{(+)2} = 0\,, \tag{3.2}$$
$$d\omega^{(+)+} + \omega^{(+)+}\omega^{(+)2} - is^{(+)}\sqrt{2}\xi^{*n}\xi_n = 0\,, \tag{3.3}$$
$$d\xi_n^{(+)} - \frac{1}{2}\omega^{(+)2}\xi_n^{(+)} = 0\,, \tag{3.4}$$

and the $(-)$ algebra reads

$$d\omega^{(-)2} = 0\,, \tag{3.5}$$
$$d\omega^{(-)-} - \omega^{(-)-}\omega^{(-)2} - is^{(-)}\sqrt{2}\zeta^{*n}\zeta_n = 0\,, \tag{3.6}$$
$$d\zeta_n^{(-)} + \frac{1}{2}\omega^{(-)2}\zeta_n^{(-)} = 0\,. \tag{3.7}$$

From the knowledge of which 1-form make the supersolvable algebra we can deduce the generic element of the supersolvable algebra (see appendix C for a complete explanation); for example for the $(+)$ algebra we get

$$g^{(+)} = \begin{pmatrix} \frac{1}{2}\alpha^{(+)2} & & \\ -\frac{i}{\sqrt{2}}\alpha^{+} & -\frac{1}{2}\alpha^{(+)2} & \tau_m \\ -s^{(+)}\tau^{*n} & & \end{pmatrix};$$

it is now an easy task to derive the explicit dependence on these coordinates of the (+) 1-forms from the generic group element $G^{(+)} = \exp\left(g_2^{(+)}\right)\exp\left(g_+^{(+)}\right)\exp\left(g_\tau^{(+)}\right)$:

$$\begin{aligned}
\omega^{(+)2} &= d\alpha^{(+)2}\,, \\
\omega^{(+)+} &= d\alpha^{+} - \alpha^{+} d\alpha^{(+)2} - \frac{i}{\sqrt{2}} s^{(+)}\left(\tau^{*n}\left(\overrightarrow{d} - \overleftarrow{d}\right)\tau_n\right), \\
\xi_n^{(+)} &= d\tau_n - \frac{1}{2}\tau_n d\alpha^{(+)2}\,,
\end{aligned}$$

Similarly for the (-) algebra we get

$$\begin{aligned}
\omega^{(-)2} &= d\alpha^{(-)2}\,, \\
\omega^{(-)-} &= d\alpha^{-} + \alpha^{-} d\alpha^{(-)2} - \frac{i}{\sqrt{2}} s^{(-)}\left(\theta^{*n}\left(\overrightarrow{d} - \overleftarrow{d}\right)\theta_n\right), \\
\zeta_n^{(-)} &= d\theta_n + \frac{1}{2}\theta_n d\alpha^{(-)2}\,.
\end{aligned}$$

In order to find the κ gauge fixed expression for the dreibein and the gravitini we have to relate the coset AdS_3 coordinates with those of the two $SO(1,2)$: this is done comparing $b^a P_a = |e| b^a J_a^{(+)} - |e| b^a J_a^{(-)}$ with $\alpha^{(+)a} J_a^{(+)} + \alpha^{(-)a} J_a^{(-)}$. In particular we get $\alpha^{(+)2} = -\alpha^{(-)2} = |e| b^2$, $\alpha^{(+)+} = |e| b^+$ and $\alpha^{(-)-} = -|e| b^-$. Using the relation $E^a = \frac{1}{2|e|}\left(\omega^{(+)a} - \omega^{(-)a}\right)$ and changing variables as $r = \exp\left(|e| b^2\right)$, $x^\pm = \frac{b^\pm}{2\,|e|\,r}$, $\theta^{(+)} = \frac{\tau}{|e|\sqrt{2\,r}}$ and $\theta^{(-)} = \frac{\theta}{|e|\sqrt{2\,r}}$ we get the final κ gauge fixed expression for the dreibein and the gravitini

$$E^{\pm} = |e| r \left(dx^{\pm} \pm \frac{i}{\sqrt{2}}\, s^{(\pm)}\, \theta^{(\pm)*n}\left(\overrightarrow{d} - \overleftarrow{d}\right)\theta_{(\pm)n}\right), \tag{3.8}$$

$$E^2 = \frac{1}{|e|}\frac{dr}{r}\,, \tag{3.9}$$

$$\chi_n^{(+)} = |e|\sqrt{2\,r}\begin{pmatrix} 0 \\ d\theta_n^{(+)} \end{pmatrix}, \tag{3.10}$$

$$\chi_n^{(-)} = |e|\sqrt{2\,r}\begin{pmatrix} d\theta_n^{(-)} \\ 0 \end{pmatrix}, \tag{3.11}$$

With these expressions in hand we try to integrate the κ gauge fixed version of the complex 3 field strength

$$G = \epsilon_{\alpha\beta} V_+^{\alpha} dA^{\beta} = G_{\widehat{a}\widehat{b}\widehat{c}} V^{\widehat{a}} V^{\widehat{b}} V^{\widehat{c}} + 2i\overline{\Psi}_c \Gamma_{\widehat{a}} \Psi V^{\widehat{a}}$$

($V_\pm^\alpha$ are constant). This is approach is possible because the operation of pull-back on the surface which fixes the gauge of the κ symmetry commutes with the external derivatives.

We notice immediately that there is a term which is closed but not exact as it is easy to see from the fact that one of the addends is the S_3 volume form . If we use

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projective coordinates for the 3-sphere,[3] the final answer is[4]

$$
\begin{aligned}
G = d\left(\epsilon_{\alpha\beta}V_+^\alpha A^\beta\right) &= \frac{2}{3}e\rho\epsilon_{ijk}E^iE^jE^k + d\left(\frac{e}{2|e|^3}\omega^{(+)+}\omega^{(-)-}\right. \\
&\left. - \frac{1}{|e|^2}e^{i(\varphi_{(+)}+\varphi_{(-)})}\overline{\chi}_N^{(+)}\chi_M^{(-)}\widetilde{\overline{\eta}}_{(s)N}\widetilde{\eta}_{(-s)M}\right) = \qquad (3.12)\\
&= \frac{16e}{3|e|^3}\rho\epsilon_{ijk}\frac{dy^i\wedge dy^j\wedge dy^k}{(1-y^2)^3} + \qquad (3.13)\\
&+d\Bigg\{-2e|e|r^2\left[dx^+ + \frac{i}{\sqrt{2}}\theta^{(+)*n}\left(\overrightarrow{d}-\overleftarrow{d}\right)\theta_{(+)n}\right]\wedge \\
&\qquad\wedge\left[dx^- + \frac{i}{\sqrt{2}}\theta^{(-)*n}\left(\overrightarrow{d}-\overleftarrow{d}\right)\theta_{(-)n}\right] + \\
&\quad + 2i\frac{e}{|e|^2}r\Big[d\theta^{(+)*n}d\theta_m^{(-)}\widetilde{\overline{\eta}}_{(s)cn}\widetilde{\eta}^m_{(-s)c} + d\theta_n^{(+)}d\theta^{(-)*m}\widetilde{\overline{\eta}}^n_{(s)}\widetilde{\eta}_{(-s)m} \\
&\quad - i\,d\theta_n^{(+)}d\theta_m^{(-)}\widetilde{\overline{\eta}}^n_{(s)}\widetilde{\eta}^m_{(-s)c} - i\,d\theta^{(+)*n}d\theta^{(-)*m}\widetilde{\overline{\eta}}_{(s)cn}\widetilde{\eta}_{(-s)m}\Big]\Bigg\}.
\end{aligned}
$$

4. The action

We have now all the necessary ingredients to be able to write down the action on this background using the general form given in [18] or the specific action derived in app. E (after setting $\phi = -\frac{\pi}{2}$):

$$
\begin{aligned}
S = \int_{\Sigma_2} d^2\xi\sqrt{-g}\,g^{\alpha\beta}\frac{1}{2}&\Bigg\{2|e|^2r^2\left[\partial_\alpha x^+ + \frac{i}{\sqrt{2}}\theta^{(+)*n}\left(\overrightarrow{\partial}_\alpha-\overleftarrow{\partial}_\alpha\right)\theta_{(+)n}\right]\times \\
&\times\left[\partial_\beta x^- + \frac{i}{\sqrt{2}}\theta^{(-)*n}\left(\overrightarrow{\partial}_\beta-\overleftarrow{\partial}_\beta\right)\theta_{(-)n}\right] - \\
&-\frac{1}{|e|^2}\frac{\partial_\alpha r\,\partial_\beta r}{r^2} - \frac{4}{|e|^2}\delta_{ij}\frac{\partial_\alpha y^i\,\partial_\beta y^j}{(1-y^2)^2} - \delta_{rs}\,\partial_\alpha z^r\,\partial_\beta z^s\Bigg\} + \\
+|e|\Im e r^2\left[dx^+ + \frac{i}{\sqrt{2}}\theta^{(+)*n}\left(\overrightarrow{d}-\overleftarrow{d}\right)\theta_{(+)n}\right]&\wedge\left[dx^- + \frac{i}{\sqrt{2}}\theta^{(-)*n}\left(\overrightarrow{d}-\overleftarrow{d}\right)\theta_{(-)n}\right] - \\
-\frac{\Re e}{|e|}r\Big[d\theta^{(+)*n}\wedge d\theta_m^{(-)}\widetilde{\overline{\eta}}_{(s)cn}\widetilde{\eta}^m_{(-s)c} &+ d\theta_n^{(+)}\wedge d\theta^{(-)*m}\widetilde{\overline{\eta}}^n_{(s)}\widetilde{\eta}_{(-s)m} -
\end{aligned}
$$

[3] Our conventions for the S_3 coset manifold with the Killing induced metric, i.e. negative definite, are ($y^2 = \eta_{ij}y^iy^j = -\delta_{ij}y^iy^j$)

$$
E^i = \frac{2}{|e|}\frac{dy^i}{1-y^2}, \qquad \varpi^{ij} = \frac{4y^{[i}dy^{j]}}{1-y^2}
$$

[4] Actually the sign of the second addend of eq . (3.12) is not fixed because there is a possible sign ambiguity in the value of $e^{i(\varphi_{(+)}+\varphi_{(-)})}$ which can be fixed as discussed in app. D.

$$
\left. - i d\theta^{(+)}_n \wedge d\theta^{(-)}_m \, \widetilde{\overline{\eta}}^{n}_{(s)} \widetilde{\eta}^{m}_{(-s)c} - i d\theta^{(+)*n} \wedge d\theta^{(-)*m} \, \widetilde{\overline{\eta}}_{(s)cn} \widetilde{\eta}_{(-s)m} \right] -
$$

$$
-\frac{1}{3}\rho \Im e \int_{B_3} \frac{8}{|e|^3} \epsilon_{ijk} \frac{dy^i \wedge dy^j \wedge dy^k}{\left(1-y^2\right)^3} \,. \tag{4.1}
$$

We notice that this action is written with the unusual metric which is mostly minus and with the use of the $AdS_{3|4}$ fermionic coordinates like in [4], we could as well have used fermionic coordinates which transform as spinors for $SO(1,2) \otimes SO(3) \otimes SO(4)$ like in [5]. This action can be rewritten in a more compact form using $6D$ pseudo-Majorana Weyl spinors when $\Im e = 0$, this is done in app. G.

It is clear from the explicit form of the action that setting $\Re e = 0$ is to choose a pure NSNS background while setting $\Im e = 0$ is equivalent to choose a pure RR background, but now the WZW term and the chiral structure disappear too. The same happened in the case treated previously, i.e. $AdS_5 \otimes S_5$. It is therefore tempting to argue that pure RR backgrounds are described by conformal actions without WZWN term. This is a further hint of fact that in order to understand pure RR backgrounds requires a lot of efforts.

This action should be conformal because in the supercosets approach there can be constructed two WZ terms [7] whose coefficients, in proper unities, must be integer, therefore they cannot mix and renormalize but the kinetic term has a relative weight fixed by the background solution hence it cannot be renormalized too. As a further evidence of this claim we notice that in the pure NSNS background the relative normalization of the kinetic term and the WZWN term is right and equal to $\frac{1}{3}$.

As derived in app. F or as it can be easily derived requiring the invertibility of the fermionic kinetic operator in the form of the action given in eq. (G.1) (it is possible to use eq. (4.1) too but the computation is longer) can only describe the states which satisfy the condition

$$
2r^2 \left(E^-_0 - \frac{\Im e}{|e|} E^-_1 \right) \left(E^+_0 + \frac{\Im e}{|e|} E^+_1 \right) + \left(\frac{\Re e}{|e|} \right)^2 \left(- \left(E^2_1 \right)^2 - \delta_{ij} E^i_1 E^j_1 \right) \neq 0 \,, \tag{4.2}
$$

where $E^{\hat{a}}_\alpha$ is the component of the pullback of $E^{\hat{a}}$ on the string surface along $d\xi^\alpha$.

In particular for the pure NSNS background it happens that the static solution (take $\Im e = -|e|$ for definiteness)

$$
x^\pm = \xi^\pm \,, \qquad r = const \,, \qquad \Theta = 0 \,, \tag{4.3}
$$

cannot be described by the action even if it is a solution of the equations of motion since it does not satisfy eq. (4.2) nevertheless the arguments of [19] do not apply since (4.3) is not a viable solution for a fundamental IIB string which has to be closed, i.e. $x^\pm$ have to be periodic in σ.

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A. Conventions

We will use the following conventions:

- Indices: $\widehat{a}, \widehat{b}, \ldots \in \{0, \ldots, 9\}$, $a, b, \ldots \in \{0, 1, 2\}$, $u, v, \ldots \in \{3, \ldots, 9\}$, $i, j, \ldots \in \{3, 4, 5\}$, $r, s, \ldots \in \{6, \ldots, 9\}$; WS indices $\alpha, \beta, \ldots \in \{0, 1\}$;
- Epsilon: $\epsilon_{0\ldots 9} = \epsilon_{012} = \epsilon_{345} = 1$; WS $\epsilon_{01} = 1$;
- metric: $\eta_{\widehat{a}\widehat{b}} = diag(+1, -1, \ldots - 1)$, $\eta_{ab} = diag(+1, -1, -1)$, $\eta_{ij} = diag(-1, -1, -1)$, $\eta_{rs} = diag(-1, -1, -1, -1)$; WS $\eta_{\alpha\beta} = diag(1, -1)$;
- lightcone coordinates for AdS_3: $x^{\pm} = \frac{1}{\sqrt{2}}(x^0 \pm x^1)$, $\epsilon_{+-2} = -1$;
- 1+9D gamma matrices

$$\begin{aligned} \{\Gamma_{\widehat{a}}, \Gamma_{\widehat{b}}\} &= 2\eta_{\widehat{a}\widehat{b}} \,, \\ \Gamma_{11} &= \begin{pmatrix} 1_{16} & \\ & -1_{16} \end{pmatrix} , \\ \Gamma^T_{\widehat{a}} &= -\widehat{C}^{-1}\Gamma_{\widehat{a}}\widehat{C} \,, \qquad \widehat{C}^T = -\widehat{C} \,, \qquad \widehat{C}^{\dagger} = \widehat{C} \,; \end{aligned}$$

- 1+2D (AdS_3) gamma matrices

$$\begin{aligned} \{\gamma_a, \gamma_b\} &= 2\eta_{ab} \,, \\ \gamma_0\gamma_1\gamma_2 &= i\, 1_2 \,, \\ \gamma^T_a &= -C^{-1}\gamma_a C \,, \qquad C^T = -C \,, \qquad C^{\dagger} = C \,, \end{aligned}$$

  explicitly we have

$$\begin{aligned} \gamma^0 &= \sigma_1 = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix} , \qquad \gamma^1 = i\sigma_2 = \begin{pmatrix} & 1 \\ -1 & \end{pmatrix} , \\ \gamma^2 &= -i\sigma_3 = \begin{pmatrix} -i & \\ & i \end{pmatrix} , \qquad C = \sigma_2 \,; \end{aligned}$$

- 0+7D ($S_3 \otimes T^4$) gamma matrices

$$\begin{aligned} \{\widetilde{\gamma}_u, \widetilde{\gamma}_v\} &= 2\eta_{uv} = -2\delta_{uv} \,, \\ \widetilde{\gamma}_3 \ldots \widetilde{\gamma}_8 &= 1_8 \,, \\ \widetilde{\gamma}^T_u &= -\widetilde{C}^{-1}\, \widetilde{\gamma}_u\, \widetilde{C} \,, \qquad \widetilde{C}^T = +\widetilde{C} \,, \qquad \widetilde{C}^{\dagger} = \widetilde{C} \end{aligned}$$

  Moreover given a $S_3 \otimes T^4$ spinor $\widetilde{\eta}$ we use the notation $\overline{\widetilde{\eta}} = \widetilde{\eta}^{\dagger}$.

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- 0+3D (S_3) gamma matrices

$$
\begin{aligned}
\{\gamma_i, \gamma_j\} &= 2\eta_{ij}\,, \\
\gamma_3\gamma_4\gamma_5 &= 1_2\,, \\
\gamma_i^T &= -C^{-1}\,\gamma_i\,C\,, \qquad C^T = -C\,, \qquad C^\dagger = C\,.
\end{aligned}
$$

Moreover given a S_3 spinor η we use the notation $\overline{\eta} = \eta^\dagger$.

- 0+4D (T^4) gamma matrices

$$
\begin{aligned}
\{\gamma_r, \gamma_s\} &= 2\eta_{rs}\,, \\
\gamma_6\gamma_7\gamma_8\gamma_9 &= \overline{\gamma}_5\,, \\
\overline{\gamma}_5^2 &= 1_4\,, & \overline{\gamma}_5^\dagger &= \overline{\gamma}_5\,, \\
\gamma_r^T &= -\overline{C}^{-1}\gamma_r\,\overline{C}\,, & \overline{\gamma}_5^T &= +\overline{C}^{-1}\,\overline{\gamma}_5\,\overline{C}\,, \\
\overline{C}^T &= -\overline{C}\,, & \overline{C}^\dagger &= \overline{C}\,.
\end{aligned}
$$

Moreover given a T^4 spinor κ we use the notation $\overline{\kappa} = \kappa^\dagger$.

- 1+9D gamma matrices expressed using 1+2D and 0+7D gamma matrices

$$
\begin{aligned}
\Gamma_a &= \gamma_a \otimes 1_8 \otimes \sigma_1\,, \\
\Gamma_u &= 1 \otimes \tilde{\gamma}_u \otimes (-\sigma_2)\,, \\
\hat{C} &= C \otimes \tilde{C} \otimes \sigma_1\,;
\end{aligned}
$$

- 0+7D gamma matrices expressed using 0+3D and 0+4D gamma matrices

$$
\begin{aligned}
\tilde{\gamma}_i &= \gamma_i \otimes \overline{\gamma}_5\,, \\
\tilde{\gamma}_r &= 1_2 \otimes \gamma_r\,, \\
\tilde{C} &= C \otimes \overline{C}\,;
\end{aligned}
$$

- 1+9D gamma matrices expressed using 1+2D , 0+3D and 0+4D gamma matrices

$$
\begin{aligned}
\Gamma_a &= \gamma_a \otimes (1_2 \otimes 1_4) \otimes \sigma_1\,, \\
\Gamma_i &= 1 \otimes (\gamma_i \otimes \overline{\gamma}_5) \otimes (-\sigma_2)\,, \\
\Gamma_r &= 1 \otimes (1_2 \otimes \gamma_r) \otimes (-\sigma_2)\,, \\
\hat{C} &= C \otimes (C \otimes \overline{C}) \otimes \sigma_1\,.
\end{aligned}
$$

B. Decomposition $so(2,2) = so(1,3) \oplus so(1,3)$ and AdS_3

We normalize the $so(2,2)$ generators as

$$
[\,M_{\mu\nu}\,,\,M_{\rho\sigma}\,] = -4\,\eta_{[\mu\ [\rho}\ M_{\nu]\ \bullet\ \sigma]}\,,
$$

where $\eta = diag(+,-,-+)$ with $\mu,\nu \in \{0,1,2,3\}$. Notice that these indices have a priori nothing to do with the spatial indices of AdS_3, but in view of the identification of AdS_3 as a coset $\frac{SO(2,2)}{SO(1,2)}$ we can write

$$\begin{aligned} P_a &= |e|\, M_{a3}\,, \\ L_{ab} &= M_{ab}\,, \end{aligned}$$

where L are the Lorentz generators, P are the translations , $|e|^2$ is the curvature and the indices $a,b,\ldots \in \{0,1,2\}$ are the tangential indices to AdS_3.

We can now define the linear combinations

$$J^{(\pm)}_{ab} = \epsilon_{ab.}^{\cdot\cdot c}\, J^{(\pm)}_c = \frac{1}{2}\left(M_{ab} \pm \frac{1}{2}\epsilon_{ab\mu\nu}\, M^{\mu\nu}\right),$$

where $\epsilon_{0123} = +1$ and easily verify that $J^{(\pm)}_c$ verify a $so(1,2)$ algebra given by

$$[\, J_a\,,\ J_b\,] = -\epsilon_{abc}\, J^c\,,$$

with $\epsilon_{012} = +1$. Since this is a change of basis we can express the M's through the J's as

$$\begin{aligned} M_{ab} &= J^{(+)}_{ab} + J^{(-)}_{ab} = \epsilon_{abc}\,\left(J^{(+)c} + J^{(-)c}\right), \\ M_{a3} &= \frac{1}{2}\epsilon_{abc}\,\left(J^{(+)bc} - J^{(-)bc}\right) = J^{(+)}_a - J^{(-)}_a\,, \end{aligned}$$

This change of basis reflects on the dual space in the following way

$$\begin{aligned} \Omega^{ab} &= \frac{1}{2}\left(\omega^{(+)ab} + \omega^{(-)ab}\right) = \frac{1}{2}\epsilon^{ab}_{\cdot\cdot c}\,\left(\omega^{(+)c} + \omega^{(-)c}\right), \\ \Omega^{a3} = |e|\, E^a &= \frac{1}{4}\epsilon^{a}_{\cdot bc}\,\left(\omega^{(+)bc} - \omega^{(-)bc}\right) = \frac{1}{2}\left(\omega^{(+)a} - \omega^{(-)a}\right), \end{aligned}$$

This can be easily obtained comparing the decompositions of a generic $so(2,2)$ valued 1-form $\mu = -\frac{1}{2}\Omega^{\mu\nu}M_{\mu\nu} = -\frac{1}{2}\Omega^{ab}L_{ab} - E^a P_a$ (the coefficient $-\frac{1}{2}$ is chosen in order to get the usual expression for the curvature when computing the Maurer-Cartan) with the direct sum of two generic $so(1,3)$ valued 1-forms $\mu = -\frac{1}{2}\omega^{(+)ab}J^{(+)}_{ab} - \frac{1}{2}\omega^{(-)ab}J^{(-)}_{ab} = -\frac{1}{2}\omega^{(+)a}J^{(+)}_a - \frac{1}{2}\omega^{(-)a}J^{(-)}_a$.

C. The $SU(1,1|2,0)$ and $SU(1,1|0,2)$ Maurer-Cartan

We define the two algebras as the set of supermatrices g satisfying

$$g^\dagger G + G g = 0\,, \qquad G = \begin{pmatrix} \gamma_0 & \\ & s\,1_2 \end{pmatrix},$$

where $s=1$ for $SU(1,1|2,0)$ and $s=-1$ for $SU(1,1|0,2)$. We can now easily write down the most general element as

$$g = \begin{pmatrix} -\alpha^a \frac{i}{2}\gamma_a & \Theta_m \\ -s\,\overline{\Theta}^n & \alpha^n_{\ .m} \end{pmatrix} + \beta 1_4\,, \qquad \alpha^{a*} = \alpha^a, \qquad (\alpha^n_{\ .m})^* = -\alpha^m_{\ .n}\ \alpha^m_m = 0\,, \qquad \beta^* = -\beta,$$

where α^a ($a \in \{0,1,2\}$), α^n_m ($m,n \in \{1,2\}$) and β are bosonic while $\Theta_n = \begin{pmatrix} \theta_n \\ \tau_n \end{pmatrix}$ are 2 3D spinors, i.e. θ and τ are Grassman variables and $\overline{\Theta}^n = \Theta^{\dagger n}\gamma_0$.

Given the previous expression for the generic element of the algebra we could compute the left invariant forms which satisfy the Maurer-Cartan equation in the standard way as $\mu(\alpha^a, \alpha^n_m, \beta, \Theta_n) = \exp(-g)\, d\exp(g)$ but, since we are interested only in the expression for the Maurer-Cartan equation, we need only the expression for a generic (super)Lie algebra valued 1-form

$$\mu = \begin{pmatrix} -\omega^a \frac{i}{2}\gamma_a & \chi_m \\ -s\,\overline{\chi}^n & A^n_{\ .m} \end{pmatrix} + B\, 1_4\,, \qquad A^\dagger = -A\,, \qquad B^* = -B\,,$$

where the 1-forms ω, B and A are bosonic while the 1-form χ is a fermionic spinorial 1-form and we have defined $\overline{\chi}^n = \chi^{\dagger n}\gamma_0$. From the usual expression for the the Maurer-Cartan equation $d\mu + \mu\mu = 0$ we deduce the Maurer-Cartan equations as

$$\begin{aligned}
&d\omega^a + \frac{1}{2}\epsilon^a_{\ .bc}\,\omega^b\omega^c - is\,\overline{\chi}^n\gamma^a\chi_n = 0\,,\\
&dA^n_{\ .m} + A^n_{\ .p}\,A^p_{\ .m} - s\left(\overline{\chi}^n\chi_m - \frac{1}{2}\delta^n_m\overline{\chi}^p\chi_p\right) = 0\,,\\
&dB - s\frac{1}{2}\overline{\chi}^p\chi_p = 0\,,\\
&d\chi_n - \frac{i}{2}\omega^a\gamma_a\chi_n + \chi_m A^m_{\ .n} = 0\,.
\end{aligned}$$

The first and last equations can be rewritten using $\omega^{ab} = \epsilon^{abc}\omega_c$ in a more standard way as

$$\begin{aligned}
&d\omega^{ab} - \omega^a_{\ .c}\omega^{cb} - s\overline{\chi}^n\gamma^{ab}\chi_n = 0\,,\\
&d\chi_n - \frac{1}{4}\omega^{ab}\gamma_{ab}\chi_n + \chi_m A^m_{\ .n} = 0\,.
\end{aligned}$$

D. Derivation of the supergravitational background

As a first step we record here two properties which turn out very important in the following

$$\overline{\eta}^1\gamma^i\eta_1 + \overline{\eta}^2\gamma^i\eta_2 = 0\,, \tag{D.1}$$
$$A^1_1 + A^2_2 = 0\,; \tag{D.2}$$

the first one is a consequence of the symplectic-Majorana condition (2.18) satisfied by η while the second equation is a consequence of the definition of A as a $SU(2)$ gauge connection.

The projection of the torsion equation on AdS_3 yields

$$T^a = \frac{1}{2|e|}\left[\left(d\omega^{(+)a} + \frac{1}{2}\epsilon^{abc}\omega_b^{(+)}\omega_c^{(+)}\right) - \left(d\omega^{(-)a} + \frac{1}{2}\epsilon^{abc}\omega_b^{(-)}\omega_c^{(-)}\right)\right] - i\overline{\Psi}\Gamma^a\Psi \tag{D.3}$$
$$= \frac{1}{2|e|}\left[is^{(+)}\overline{\chi}^{(+)n}\gamma^a\chi_n^{(+)} - is^{(-)}\overline{\chi}^{(-)n}\gamma^a\chi_n^{(-)}\right] - 2i\kappa_3^2\left[\overline{\chi}^{(+)n}\gamma^a\chi_n^{(+)} + \overline{\chi}^{(-)n}\gamma^a\chi_n^{(-)}\right],$$

where we have used (2.20) to eliminate $\omega^{(\pm)}$ and we have imposed

$$e^{2i(\varphi_{(-)}-\varphi_{(+)})} = -1 \tag{D.4}$$

in order to eliminate the mixed gravitini current $\overline{\chi}^{(+)}\chi^{(-)}$in the current $\overline{\Psi}\Gamma^a\Psi$. We notice that eq. (2.37) is not the only solution of eq. (D.4), the other solution with the r.h.s. of eq. (2.37) equal to $-i$ does not yield a different solution since the sign can be reabsorbed in the field redefinition $\chi^{(-)} \to -\chi^{(-)}$.

From eq. (D.4) we can now easily derive the conditions

$$s^{(\pm)} = \pm 1\,, \tag{D.5}$$
$$\kappa_3 = \frac{1}{2\sqrt{|e|}}\,, \tag{D.6}$$

The first condition is quite important because it means that $SU(1,1|2)^2$ is *not* the proper global symmetry of this background.

We can now turn to exam the projection of the torsion on S_3; when we use

$$s = -t\,,$$
$$\kappa_2^{(\pm)} = \mp s|e|\kappa_1^{(\pm)}\,,$$

in the expression of the torsion we can eliminate the sectors $A^{(\pm)}E^i$ and $A^{(+)}A^{(-)}$ and we are left with

$$\begin{aligned} T^i = {} & \kappa_1^{(+)}\left[\tilde{\overline{\eta}}{}^m_{(s)}\,\tilde{\gamma}^i\,\tilde{\eta}_{(s)n}\,dA^{(+)n}_{\ .\,m} - \kappa_2^{(+)}\tilde{\overline{\eta}}{}^m_{(s)}\,\tilde{\gamma}^{ij}\,\tilde{\eta}_{(s)n}\,\tilde{\overline{\eta}}{}^p_{(s)}\,\tilde{\gamma}_j\,\tilde{\eta}_{(s)q}\,A^{(+)n}_{\ .\,m}\,A^{(+)q}_{\ .\,p}\right] + \\ & +\kappa_1^{(-)}\left[\tilde{\overline{\eta}}{}^m_{(t)}\,\tilde{\gamma}^i\,\tilde{\eta}_{(t)n}\,dA^{(-)n}_{\ .\,m} - \kappa_2^{(-)}\tilde{\overline{\eta}}{}^m_{(t)}\,\tilde{\gamma}^{ij}\,\tilde{\eta}_{(t)n}\,\tilde{\overline{\eta}}{}^p_{(t)}\,\tilde{\gamma}_j\,\tilde{\eta}_{(t)q}\,A^{(-)n}_{\ .\,m}\,A^{(-)q}_{\ .\,p}\right] + \\ & -i\overline{\Psi}\Gamma^i\Psi\,; \end{aligned}$$

now, in order to be able to use eq. (2.21) to eliminate the A in favor of χ, we need a Fierz identity like the first one of the two identities

$$\tilde{\gamma}^j\,\tilde{\eta}_{(r)n}\,\tilde{\overline{\eta}}{}^p_{(r)}\,\tilde{\gamma}_j\,\tilde{\eta}_{(r)q} = 2\kappa_4\,\delta^p_n\,\tilde{\eta}_{(r)q} + \delta^p_q\,(\ldots) \tag{D.7}$$
$$\tilde{\gamma}^j\,\tilde{\eta}^n_{(r)c}\,\tilde{\overline{\eta}}{}^p_{(r)}\,\tilde{\gamma}_j\,\tilde{\eta}_{(r)q} = 2\kappa_5\,\delta^n_q\,\tilde{\eta}^p_{(r)c} + \delta^p_q\,(\ldots) \tag{D.8}$$

which have to be valid for $r = s, t$ and where $(\ldots)$ stands for something of which it does not matter to know the actual value (as far as it is independent of p, q) in view of eq. (D.2); we notice moreover that the second equation (D.8) turns out to be necessary for verifying the gravitino Bianchi identity. These equations (D.7), (D.8) can be rewritten in terms of H_n (2.28) as

$$\begin{aligned} H_q H^{\dagger p} H_n &= \kappa_4\, \delta^p_n\, H_q + \kappa_6\, \delta^p_q\, H_n\,, \\ H_q H^{\dagger p} H^n_c &= \kappa_5\, \delta^n_q\, H^p_c + \kappa_7\, \delta^p_q\, H^n_c\,, \end{aligned}$$

where we have filled the $(\ldots)$ with the only objects with the right transformation properties that could be used. We have moreover defined H^n_c to be the matrices entering eq. (2.28) in place of H^n when we define η^n_c; explictly $\sigma_2 H^{*n} \sigma_2 = H^n_c$.

Since these equations are cubic and therefore difficult to solve it is better to try to find further constraints on the matrices H_N (2.16); some constraints come from the ansatz that the current $\overline{\Psi}\Gamma^i\Psi$ be proportional to $\overline{\chi}^{(+)m}\chi^{(+)}_n\, \overline{\eta}^n_{(s)}\gamma^i\eta_{(s)m} + \overline{\chi}^{(-)m}\chi^{(-)}_n\, \overline{\eta}^n_{(t)}\gamma^i\eta_{(t)m}$ without currents like $\overline{\chi}^n\chi^m_c$:

$$\left(H^m_c H^\dagger_{cn} - H_n H^{\dagger m}\right)_{pq} = a(\delta^m_p\delta^q_n - \epsilon_{np}\epsilon^{mq}) + c^m_n\delta^p_q\,, \tag{D.9}$$

$$H_m H^\dagger_{cn} - H_n H^\dagger_{cm} = c_{nm} 1\,. \tag{D.10}$$

To these constraints we have to add the normalization conditions (2.17), (2.18), (2.29), which in this language read:

$$tr(H^{\dagger n} H_m) = \delta^n_m\,, \tag{D.11}$$

$$tr(H^{\dagger n} H^m_c) = 0\,. \tag{D.12}$$

As a consequence of eqs. (D.11), (D.12) and eq. (D.9) we deduce that

$$H_1 H^{\dagger 1} = \frac{1}{2}1_2 - \frac{1}{2}a\sigma_3\,, \tag{D.13}$$

$$H_2 H^{\dagger 2} = \frac{1}{2}1_2 + \frac{1}{2}a\sigma_3\,, \tag{D.14}$$

$$H_1 H^{\dagger 2} = -a\sigma_- = -a(\sigma_1 - i\sigma_2)\,, \tag{D.15}$$

$$H_2 H^{\dagger 1} = -a\sigma_+\,. \tag{D.16}$$

Using these equations (D.13)–(D.16) into (D.7), (D.8) we get a over constrained system whose solution is :

$$\begin{gathered} a = 1\,, \qquad \kappa_4 = 1\,, \qquad \kappa_5 = -1\,, \qquad \kappa_6 = 0\,, \qquad \kappa_7 = 1\,, \\ H_1 = \begin{pmatrix} 0 & 0 \\ -h_2 & h_1 \end{pmatrix}, \qquad H_2 = \begin{pmatrix} h_2 & -h_1 \\ 0 & 0 \end{pmatrix}, \\ H^1_c = \begin{pmatrix} h^*_1 & h^*_2 \\ 0 & 0 \end{pmatrix}, \qquad H^2_c = \begin{pmatrix} 0 & 0 \\ h^*_1 & h^*_2 \end{pmatrix}, \end{gathered}$$

with $|h_1|^2 + |h_2|^2 = 1$. The general solution can be obtained by a $SU(2)$ rotation applied to (H_n) thought as a two component complex vector whose entries are matrices.

We can now perform a $USp(2)$ rotation on η and κ (which leaves invariant the symplectic-Majorana and normalization conditions) to bring the previous H to the ones given in the text (2.38).

E. The GS superstring action on $AdS_3 \otimes S_3 \otimes T^4$ in first order formalism and its κ symmetry

It is purpose of this appendix to determine the explicit form of the superstring action propagating on $AdS_3 \otimes S_3 \otimes T^4$ and the κ symmetry transformation rules for its fields. This can be done in two different ways either starting in $D = 10$ and then compactifying or directly in $D = 3$.

We start with the former approach writing an ansatz inspired by the knowledge of the second order action [18] using the notations of [15] and from the fact that the scalars are constant:

$$S = \int_\Sigma c_1 \Pi^{\widehat{a}}_\alpha \, i_* V^{\widehat{b}} \wedge e^\beta \, \eta_{\widehat{a}\widehat{b}} \, \epsilon_{\alpha\beta} + c_2 \Pi^{\widehat{a}}_\alpha \Pi^{\widehat{b}}_\beta \, \eta_{\widehat{a}\widehat{b}} \, \eta^{\alpha\beta} \, \epsilon_{\gamma\delta} \, e^\gamma \wedge e^\delta + \\ + c_3 V_+ * A - c_3^* V_- * A \tag{E.1}$$

where $V_\pm * A = \epsilon_{\alpha\beta} V^\alpha_\pm A^\beta$, $\alpha, \beta, \ldots \in \{0, 1\}$ are worldsheet indexes and i_* is the pullback on the string worldsheet Σ due to the superimmersion $i : \Sigma \to AdS_3 \otimes S_3 \otimes T^4$.

The relative coefficient of the first two terms of the first line in eq. (E.1) is determined to be

$$c_2 = -\frac{1}{4} c_1 \,, \tag{E.2}$$

by the request that the $\Pi^{\widehat{a}}_\alpha$ equation of motion yields

$$\frac{\delta S}{\delta \Pi} = 0 \Longrightarrow i_* V^{\widehat{a}} = \Pi^{\widehat{a}}_\alpha e^\alpha \,. \tag{E.3}$$

The zweibein equation of motion gives the Virasoro constraints:

$$\frac{\delta S}{\delta e} = 0 \Longrightarrow \Pi_\alpha \bullet \Pi_\beta = \frac{1}{2} \eta_{\alpha\beta} \Pi^2 \,. \tag{E.4}$$

From the request of the action (E.1) (in the 1.5 formalism) to have a κ symmetry we determine the relative value of the coefficients of last two terms in eq. (E.1) with respect the first term in the first line:

$$c_3 = -\frac{1}{4} e^{i\phi} c_1 \,, \qquad \forall \phi \tag{E.5}$$

and the vector field $\overrightarrow{\epsilon} = \delta_\kappa\Theta\,\overrightarrow{\partial_\Theta} + \delta_\kappa\Theta_c\,\overrightarrow{\partial_{\Theta_c}} + \delta_\kappa x\,\overrightarrow{\partial_x}$ associated to the κ symmetry and the variation of the zweibein:

$$\delta_\kappa e^\alpha = 2ic_1\left(\overline{\widehat{\kappa}}^\alpha\Psi + \overline{\widehat{\kappa}_c}^\alpha\Psi_c\right).$$

The vector field $\overrightarrow{\epsilon}$ is defined by the properties

$$\underset{\overrightarrow{\epsilon}}{} \mid \psi = \widehat{\epsilon} = \Pi^{\widehat{a}}_\alpha\,\Gamma_{\widehat{a}}\widehat{\kappa}^\alpha\,, \qquad \underset{\overrightarrow{\epsilon}}{} \mid V^{\hat{a}} = 0\,, \tag{E.6}$$

$$\widehat{\kappa}_{c\alpha} = ie^{i\phi}\epsilon_{\alpha\beta}\,\widehat{\kappa}_\beta\,, \tag{E.7}$$

and its action by mean of the Lie derivative gives the supersymmetric variations (up to local Lorentz and gauge transformations) of the other fields

$$\delta_\kappa V^{\widehat{a}} = -i\,\left(\overline{\Psi}\Gamma^{\hat{a}}\widehat{\epsilon} - \overline{\Psi_c}\Gamma^{\hat{a}}\widehat{\epsilon}_c\right), \tag{E.8}$$

$$\delta_\kappa(V_+ * A) = -4i\,\overline{\Psi}\,\Gamma^{\hat{a}}\widehat{\epsilon}V_{\hat{a}}\,, \tag{E.9}$$

We have now to rewrite these expressions using the fields which we used in the rest of this paper and are proper for treating the background; to this purpose inspired by the expression for the gravitini eq. (2.26) we write

$$\widehat{\epsilon} = \kappa_3\,\begin{pmatrix} 0 \\ e^{i\varphi_{(+)}}\epsilon^{(+)}_{(n,m)}(x,\Theta)\otimes\widetilde{\eta}^{(n,m)}_{(s)} + e^{i\varphi_{(-)}}\epsilon^{(-)}_{(n,m)}(x,\Theta)\otimes\widetilde{\eta}^{(n,m)}_{(t)}\end{pmatrix} \tag{E.10}$$

$$\hat{\kappa} = \kappa_3\,\begin{pmatrix} e^{i\varphi_{(+)}}\kappa^{(+)}_{(n,m)}(x,\Theta)\otimes\widetilde{\eta}^{(n,m)}_{(s)} + e^{i\varphi_{(-)}}\kappa^{(-)}_{(n,m)}(x,\Theta)\otimes\widetilde{\eta}^{(n,m)}_{(t)} \\ 0\end{pmatrix} \tag{E.11}$$

where we impose Majorana conditions on $\epsilon^{(\pm)}_N$ and antiMajorana on $\kappa^{(\pm)}_N$ because of the presence of a Γ in eq. (E.6). In order to check the validity of this ansatz we insert these expressions in the susy transformation rules (E.8)–(E.9) and we get the same susy transformation rules for the $AdS_3\otimes S_3\otimes T^4$ fields as those which can be derived directly from the curvatures (2.20)–(2.23) and from the expressions (2.30)–(2.32):

$$\delta V^a = -\frac{i}{2\,|e|}\,\left(\overline{\chi}^{(+)}_N\gamma^a\epsilon^{(+)}_N + \overline{\chi}^{(-)}_N\gamma^a\epsilon^{(-)}_N\right), \tag{E.12}$$

$$\delta V^i = \frac{1}{2\,|e|}\,\left(\overline{\chi}^{(+)}_N\epsilon^{(+)}_M K^i_{(s)MN} + \overline{\chi}^{(-)}_N\epsilon^{(-)}_M K^i_{(t)MN}\right), \tag{E.13}$$

$$\delta V^r = 0\,, \tag{E.14}$$

$$\begin{aligned}\delta(V_+ * A) = &-\frac{i}{|e|}\,\left(e^{2i\phi_{(+)}}\overline{\chi}^{(+)}_N\gamma^a\epsilon^{(+)}_N + e^{2i\phi_{(-)}}\overline{\chi}^{(-)}_N\gamma^a\epsilon^{(-)}_N\right)\,V_a - \\ &-\frac{i}{2\,|e|}e^{i\left(\phi_{(+)}+\phi_{(-)}\right)}\,\left(\overline{\chi}^{(+)}_N\gamma^a\epsilon^{(-)}_M + \overline{\chi}^{(-)}_N\gamma^a\epsilon^{(+)}_M\right)\,J_{NM}\,V_a + \\ &+\frac{1}{|e|}\,\left(e^{2i\phi_{(+)}}\overline{\chi}^{(+)}_N\epsilon^{(+)}_M K^i_{(s)MN} + e^{2i\phi_{(-)}}\overline{\chi}^{(-)}_N\epsilon^{(-)}_M K^i_{(t)MN}\right)\,V_i + \\ &+\frac{1}{|e|}e^{i\left(\phi_{(+)}+\phi_{(-)}\right)}\,\left(\overline{\chi}^{(+)}_N\epsilon^{(-)}_M + \overline{\chi}^{(-)}_N\epsilon^{(+)}_M\right)\,J^i_{NM}\,V_i\,,\end{aligned} \tag{E.15}$$

where we have defined the following quantities:

$$
\begin{aligned}
J_{NM} &= \tilde{\bar{\tilde{\eta}}}^{N}_{(s)}\tilde{\eta}^{N}_{(t)}\,, & J^{i}_{NM} &= \tilde{\bar{\tilde{\eta}}}^{N}_{(s)}\tilde{\gamma}^{i}\tilde{\eta}^{N}_{(t)}\,,\\
K^{i}_{(s)MN} &= \tilde{\bar{\tilde{\eta}}}^{N}_{(s)}\tilde{\gamma}^{i}\tilde{\eta}^{N}_{(s)}\,, & K^{i}_{(t)MN} &= \tilde{\bar{\tilde{\eta}}}^{N}_{(t)}\tilde{\gamma}^{i}\tilde{\eta}^{N}_{(t)}\,,
\end{aligned}
$$

which enjoy the useful properties

$$
\begin{aligned}
&J_{PM}\,J_{PN} = J_{MP}\,J_{NP} = \delta_{MN}\,,\\
&K^{i}_{(s)MP}\,J_{PN} = K^{i}_{(t)MN}\,,
\end{aligned}
$$

Before we can extract the κ symmetry transformations for the $AdS_3 \otimes S_3 \otimes T^4$ fields, it is important to realize that $\tilde{\eta}^{N}_{(s)}$and $\tilde{\eta}^{N}_{(t)}$ are *not* linearly independent since they both satisfy eq. (2.12) and there are at most 4 spinors which are linearly independent over C. Both of them satisfy the Fierz identity

$$
\tilde{\eta}^{N}_{(s)}\tilde{\bar{\tilde{\eta}}}^{N}_{(s)} = \tilde{\eta}^{N}_{(t)}\tilde{\bar{\tilde{\eta}}}^{N}_{(t)} = 1_2 \otimes \frac{1+\rho\overline{\gamma}_5}{2}\,, \tag{E.16}
$$

since both $\tilde{\eta}^{N}_{(s)}$ and $\tilde{\eta}^{N}_{(t)}$ are orthonormal as follows from eq. (2.29).

We are now ready to insert eqs (E.10), (E.11) into eq. (E.6) in order to get the relation between the parameters ϵ_N of the supersymmetry variation and the parameters κ_N:

$$
\epsilon^{(+)}_{N} = \Pi^{a}_{\alpha}\,\gamma_a\kappa^{(+)\alpha}_{N} - i\,\Pi^{i}_{\alpha}\,K^{i}_{(s)NM}\,\kappa^{(+)\alpha}_{M}\,, \tag{E.17}
$$

$$
\epsilon^{(-)}_{N} = \Pi^{a}_{\alpha}\,\gamma_a\kappa^{(-)\alpha}_{N} - i\,\Pi^{i}_{\alpha}\,K^{i}_{(t)NM}\,\kappa^{(-)\alpha}_{M}\,, \tag{E.18}
$$

and into eq. (E.7) in order to get the constraints on the κ parameters:

$$
\kappa^{(+)}_{N\alpha} + e^{i\left(\phi_{(+)}-\phi_{(-)}\right)}J_{NM}\kappa^{(-)}_{M\alpha} = e^{i\left(\phi+2\phi_{(+)}\right)}\,\epsilon_{\alpha\beta}\left(\kappa^{(+)}_{N\beta} + e^{i\left(\phi_{(-)}-\phi_{(+)}\right)}J_{NM}\kappa^{(-)}_{M\beta}\right). \tag{E.19}
$$

We could have proceeded directly in $D=3$ and imposed that eq. (E.1) were invariant under the susy transformation rules (E.12)–(E.15) with properly restricted parameters $\epsilon^{(\pm)}_{N}$, then starting from an ansatz like eqs. (E.17)–(E.18) with the coefficients undetermined and imposing the cancellation of the terms proportional to $\Pi^a\Pi^i$ we would have fixed these undetermined coefficients and found a constraint equivalent to (E.19).

F. Fixing the κ symmetry with the supersolvable projector

In this appendix we want to show that it is possible to fix the κ symmetry using the supersolvable algebra and find which states can be described in this gauge: this is completely analogous to what happens in the more usual GS formulation of the superstring in $D=10$ where only the states with $p^+ \neq 0$ can be described.

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As a first step in this direction we introduce the projectors

$$P = \frac{1}{\sqrt{2}}\gamma^0\gamma^- = \frac{1+i\gamma^2}{2}\,,$$
$$Q = \frac{1}{\sqrt{2}}\gamma^0\gamma^+ = \frac{1+i\gamma^2}{2}$$

and we notice that the request of fixing the κ symmetry by using the supersolvable algebra is equivalent to showing the possibility of setting $P\psi^{(+)} = Q\psi^{(-)} = 0$, and this can be reformulated as the fact that we can find a solution for the κ symmetry parameter $\kappa^{(\pm)}_{N\alpha}$ such that we can obtain any $P\epsilon^{(+)}$ and $Q\epsilon^{(-)}$.

In order to show this we need introduce some notation for writing the following expression in a more concise way; we define therefore

$$\begin{aligned} C &= \cos(\phi + 2\phi_{(+)}) = -\frac{\Re(ee^{i\phi})}{|e|}\,, \\ S &= \sin(\phi + 2\phi_{(+)}) = -\frac{\Im(ee^{i\phi})}{|e|} \\ \lambda^{(-)}_{N\alpha} &= \kappa^{(+)}_{M\alpha} J_{MN}\,, \\ \eta^{(-)}_{N\alpha} &= \epsilon^{(+)}_{M\alpha} J_{MN}\,. \end{aligned}$$

Using this notation eq. (E.19) implies that not all the components of $\kappa^{(-)}_\alpha$ and $\lambda^{(-)}_\alpha$are independent, i.e. (from now on we drop all the ${}^{(-)}$ in order to have a lighter notation)

$$\begin{aligned} -\kappa_{N1} &= C\kappa_{N0} + S\lambda_{N0}\,, \\ -\lambda_{N1} &= -C\lambda_{N0} + S\kappa_{N0}\,. \end{aligned} \tag{F.1}$$

The equations which are to be solved and relate $P\eta$ and $Q\epsilon$ to κ and λ can be derived from (E.17,E.18) read

$$Q\epsilon_N = \Pi^p_\alpha\,\gamma_p P\kappa^\alpha_N + \left(\Pi^2_\alpha\,\gamma_2 - i\,\Pi^i_\alpha\,K^i_{(t)MN}\right)\,Q\kappa^\alpha_M \tag{F.2}$$
$$P\eta_N = \Pi^p_\alpha\,\gamma_p Q\lambda^\alpha_N + \left(\Pi^2_\alpha\,\gamma_2 - i\,\Pi^i_\alpha\,K^i_{(t)MN}\right)\,P\lambda^\alpha_M \tag{F.3}$$

and when we set $P\kappa = Q\lambda = 0$ and utilize the constraints (F.1) they can be rewritten as

$$\begin{pmatrix} Q\epsilon_N \\ P\lambda_N \end{pmatrix} = \begin{pmatrix} (\Pi^p_0\gamma_p + C\Pi^p_1\gamma_p)\,Q\delta_{NM} & S\left(\Pi^2_1\gamma_2\delta_{NM} - i\Pi^i_\alpha K^i_{(t)MN}\right)P \\ S\left(\Pi^2_1\gamma_2\delta_{NM} - i\Pi^i_\alpha K^i_{(t)MN}\right)Q & (\Pi^p_0\gamma_p - C\Pi^p_1\gamma_p)\,P\delta_{NM} \end{pmatrix} \begin{pmatrix} P\kappa_{M0} \\ Q\lambda_{M0} \end{pmatrix}$$

When we compute the determinant of the matrix entering the previous expression on the subspace where $Q = P = 1$ and we require it to be different from zero we get the desired constraint on the states and the proof that we can actually fix the κ symmetry with the supersolvable gauge which can be described by this gauge

$$2\left(\Pi^+_0 - C\Pi^+_1\right)\left(\Pi^-_0 + C\Pi^-_1\right) + S^2\left(\Pi^2_1\Pi_{21} + \Pi^i_1\Pi_{i1}\right) \neq 0\,. \tag{F.4}$$

We notice that the request $P\kappa = Q\lambda = 0$ is not restrictive at all and whichever solution to the eqs (F.2)–(F.3) implies eq. (F.4).

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G. Another useful expression for the GS action

We want now find a different expression for the action (4.1) where we make use of 6D pseudo-Majorana Weyl spinors [20]. To this purpose we construct the 6D gamma matrices as

$$\Gamma_a = \gamma_a \otimes 1 \otimes \sigma_1 \,, \qquad \Gamma_i = 1 \otimes \gamma_i \otimes \sigma_2 \,, \qquad \mathcal{C} = C \otimes C \otimes \sigma_1 \,,$$
$$\Gamma_7 = -\Gamma_0 \ldots \Gamma_5 = 1 \otimes 1 \otimes \sigma_3 \,,$$

which satisfy the properties

$$\mathcal{C}^T = \mathcal{C} \,, \qquad \Gamma_\bullet^T = -\mathcal{C}\Gamma_\bullet \mathcal{C} \,.$$

To proceed further we need the explicit expression of the S_3 spinors which satisfy eq. (2.19):

$$\eta^p_{(s)} = \frac{1}{\sqrt{1-y^2}} \left(1 + s\rho\, y^i\, \gamma_i\right) \epsilon^p \,,$$

where ϵ^p is a set of 2 constant spinors normalized as $\overline{\epsilon}_q \epsilon^p = \delta^p_q$. This can be derived as in [6].

With these definitions we can verify, using the second equation of (2.16), that the following 6D spinors

$$\Theta_p = (\theta^{(+)}_N + \theta^{(-)}_N) \otimes \epsilon^r\, H^N_{rp} \otimes \xi \,,$$

where $\xi = \begin{pmatrix} i \\ 0 \end{pmatrix}$ are a couple of pseudo-Majorana Weyl spinors; explicitly we have

$$\Theta_{cp} \equiv \mathcal{C}\overline{\Theta}^T_p = \epsilon_{pq}\, \Theta_q \,, \qquad \Gamma_7 \Theta_p = +\Theta_p \,,$$

We notice that in performing the sum $\theta^{(+)}_N + \theta^{(-)}_N$ we do not loose any d.o.f since $\sigma_3 \theta^{(\pm)}_N = \pm \theta^{(\pm)}_N$.

With the further change of bosonic variables ($y^2 = -\delta_{ij} y^i y^j$)

$$Y^2 = r \frac{1+y^2}{1-y^2} \,, \qquad Y^i = (-s\rho)\, r \frac{2y^i}{1-y^2} \,,$$

we can now easily rewrite the action as follows ($(Y)^2 = -\left(Y^2\right)^2 - \left(Y^i\right)^2$):

$$\begin{aligned} S = \int_{\Sigma_2} d^2\xi \,\sqrt{-g}\, g^{\alpha\beta} \frac{1}{2} \Big\{ &- 2|e|^2\ (Y)^2\ \left[\partial_\alpha x^+ + \frac{i}{2} \overline{\Theta}^p \Gamma^+ \partial_\alpha \Theta_p\right] \times \\ &\times \left[\partial_\beta x^- + \frac{i}{2} \overline{\Theta}^p \Gamma^- \partial_\beta \Theta_p\right] - \frac{1}{|e|^2} \delta_{ut} \frac{\partial_\alpha Y^t \partial_\beta Y^u}{(Y)^2} - \delta_{rs}\, \partial_\alpha z^r \partial_\beta z^s \Big\} - \\ &- |e| \Im e\ (Y)^2\ \left[dx^+ + \frac{i}{2} \overline{\Theta}^p \Gamma^+ d\Theta_p\right] \wedge \left[dx^- + \frac{i}{2} \overline{\Theta}^p \Gamma^- d\Theta_p\right] + \\ &+ \frac{e}{2\mid e\mid} Y^t i d\overline{\Theta}^p \Gamma_t d\Theta_p + S_{WZWN} \,, \end{aligned} \qquad \text{(G.1)}$$

where $t, u = 2, \ldots, 5$ can be interpreted as the directions transverse to the (p, q) D1 brane generating this background in the near horizon limit when we forget the T^4 σ model.

References

[1] J. Maldacena, *The large N limit of superconformal field theory and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [hep-th/9711200].

[2] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, *Phys. Lett.* **B 428** (1998) 105 [hep-th/9802109].

[3] E. Witten, *Anti de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [hep-th/9802150].

[4] I. Pesando, *A κ Gauge Fixed Type IIB Superstring Action on $AdS_5 \times S_5$*, *J. High Energy Phys.* **01** (1998) 002 [hep-th/9808020].

[5] R. Kallosh and J. Rahmfeld, *The GS string action on $AdS_5 \times S_5$*, *Phys. Lett.* **B 443** (1998) 143 [hep-th/9808038].

[6] I. Pesando, *All roads lead to Rome: supersolvable and supercosets*, hep-th/9808146.

[7] R.R Metsaev and A.A. Tseytlin, *Type IIB superstring action in $AdS_5 \times S_5$ background*, *Nucl. Phys.* **B 533** (1998) 109 [hep-th/9805028].

[8] R. Kallosh, J. Rahmfeld and A. Rajaraman, *Near horizon superspace*, *J. High Energy Phys.* **09** (1998) 002 [hep-th/9805217].

[9] R. Kallosh, *Superconformal actions in killing gauge*, hep-th/9807206.

[10] R. Kallosh and A. Rajaraman, *Vacua of M-theory and string theory*, *Phys. Rev.* **D 58** (1998) 125 [hep-th/9805041].

[11] R. Kallosh and A.K Tseytlin, *Simplifying superstring action on $AdS_5 \times S_5$*, *J. High Energy Phys.* **10** (1998) 016 [hep-th/9808088].

[12] D.Z. Freedman, S.D. Mathur, A. Matusis and L. Rastelli, *Comments on 4-point functions in CFT/AdS correspondence*, hep-th/9808006.

[13] J.H. Brodie and M. Gutperle, *String corrections to four point functions in the CFT/AdS correspondence*, *Phys. Lett.* **B 445** (1999) 296 [hep-th/9809067].

[14] A. Giveon, D. Kutazov and N. Seiberg, *Comments on string theory on AdS_3*, *Adv. Theor. Math. Phys.* **2** (1998) 733 [hep-th/9806194].

[15] L. Castellani and I. Pesando, *The complete superspace action of chiral $D = 10$ $N = 2$ supergravity*, *Int. J. Mod. Phys.* **A 8** (1993) 1125;
L. Castellani, *Chiral $D = 10$, $N = 2$ supergravity on the group manifold: 1. Free differential algebra and solution of Bianchi identities*, *Nucl. Phys.* **B 294** (1987) 877.

[16] L. Castellani, A. Ceresole, R. D'Auria, S. Ferrara, P. Fré and M. Trigiante, *G/H M-branes and AdS_{p+2} geometries*, *Nucl. Phys.* **B 527** (1998) 142 [hep-th/9803039].

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[17] G. Dall'Agata, D. Fabbri, C. Fraser, P. Fré, P. Termonia and M. Trigiante, *The $Osp(8|4)$ singleton action from the supermembrane*, hep-th/9807115.

[18] M.T. Grisaru, P. Howe, L. Mezincescu, B.E.W. Nilsson and P.K. Townsend, *$N = 2$ superstrings in a supergravity background*, *Phys. Lett.* **B 162** (1985) 116.

[19] P. Pasti, D. Sorokin and M. Tonin, *On gauge-fixed superbrane actions in AdS super-backgrounds*, hep-th/9809213.

[20] J. Rahmfeld and A. Rajaraman, *The GS string action on $AdS_3 \times S^3$ with Ramond-Ramond charge*, hep-th/9809164;
J. Park and S. Rey, *Green-Schwarz superstring on $AdS_3 \times S^3$*, *J. High Energy Phys.* **01** (1999) 001 [hep-th/9812062].